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ANALYTICAL TOOLS

Module 008

Impacts of Policies on Poverty

Axioms for Poverty Measurement

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Axioms for Poverty Measurement

by

Lorenzo Giovanni Bellù, Agricultural Policy Support Service, Policy Assistance Division, FAO, Rome, Italy

Paolo Liberati, University of Urbino, "Carlo Bo", Institute of Economics, Urbino, Italy

for the

Food and Agriculture Organization of the United Nations, FAO

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1 SUMMARY

This module illustrates on what basis a poverty index should be chosen. In particular, it discusses the desirable properties that a **poverty index** should respect. These desirable properties are called **axioms**. Four classes of axioms will be discussed:

- **focus axioms**
- **monotonicity axioms**
- **transfer axioms**
- **symmetry axioms**

As users will see, there is no single measure of poverty that respects all axioms at the same time. Choosing a poverty measure may therefore imply a trade-off between different aims.

2 INTRODUCTION

The main aim of this module is to investigate which poverty measures respect which axioms. Axioms, in this case, are mild assumptions on how a poverty measure should behave. This module will help users choose among the many poverty measures described in other modules. This topic is particularly important in policy work, as the use of different poverty measures may entail contradictory results on poverty levels. This method will give users a basis to select among poverty indeces according to pre-determined criteria.

Target audience

The target audience is that of applied analysts who want to work on poverty issues properly. Users should be familiar with basic mathematics and statistics, with a notion of income distribution and of poverty measurement.

Required background

The trainer should verify that the audience is familiar with the concept of income distribution and with the concept of poverty definition and measurement. A consolidated knowledge of poverty measures is also necessary. Basic knowledge of mathematics and statistics is required.

This modules also belongs to a set of modules that discuss poverty measurement. A complete set links of other related EASYPol modules are included at the end of this module. However, users will also find links to related material throughout the text where relevant[1].

[1] EASYPol hyperlinks are shown in blue, as follows:
a) training paths are shown in **underlined bold font**;
b) other EASYPol modules or complementary EASYPol materials are in ***bold underlined italics***;
c) links to the glossary are in **bold**; and
d) external links are in *italics*.

3 CONCEPTUAL BACKGROUND

3.1 Focus axioms

This family of axioms requires a poverty index to be insensitive to incomes above the poverty line. This is a natural consequences of the fact that only people below the poverty line count in poverty analysis. Therefore, poverty indexes are not concerned about what happens above the poverty line. There are two versions of this axiom:

Standard focus axiom (F) – Given two income distributions of the **same size** where the incomes of the poor are the same in both cases, the poverty index measured on either distribution should give the same value.

Generalised focus axiom (GF) – Given two income distributions of **different sizes** where the incomes of the poor are the same in both cases, the poverty index measured on either distribution should give the same value.

3.2 Monotonicity axioms

This class of axioms requires that if the income of a poor individual who is below the poverty line increases, the poverty index should decrease. Also in this case there are two versions:

Strong monotonicity axiom (SM) – A poverty index should decrease whenever the income of a poor individual rises.

Weak monotonicity axiom (WM) – A poverty index should decrease whenever the income of a poor individual rises, **provided that this individual remains poor**.

There is little difference but it is important to distinguish. When the income of a poor individual increases, WM requires the poverty index to decrease only if the poor person is not lifted out of poverty after the increase. SM requires the poverty index to decrease also in the case that the poor individual is lifted out of poverty. Therefore, SM implies WM.

3.3 Transfer axioms

This class of axioms is very important in poverty measure. In its general form, it requires a poverty measure to decrease after a progressive transfer and to increase after a regressive transfer. Four versions of this simple principle can be identified:

Minimal transfer axiom (MT) – This requires that the poverty index decrease (increase) after a progressive (regressive) transfer among two poor individuals remaining poor after the transfer, i.e. with the same number of poor people before and after the transfer.

Weak transfer axiom (WT) – This requires that the poverty index decrease (increase) after a progressive (regressive) transfer from an individual either above or below the

poverty line to a relatively poorer individual. Also in this case the number of poor people must be the same before and after the transfer.

Strong upward transfer axiom (SUT) – This requires that the poverty index decrease (increase) after a progressive (regressive) transfer when the poorest of the two individuals is poor both before and after the transfer and the richer of the two individuals may be either poor or non-poor as a result of the transfer.

Strong downward transfer axiom (SDT) – This requires that the poverty index decrease (increase) after a progressive (regressive) transfer from a relatively richer person, who may or may not be poor, to a poor person who may become non-poor after the transfer.

Some explanation is needed here to catch the differences between the different versions of the transfer axiom. MT simply requires that both persons involved in the transfer be poor before and after the transfer. There is no crossing of the poverty line. WT requires that only the recipient of the transfer be poor before and after the transfer, but the donor cannot cross the poverty line. SUT allows the donor to cross the poverty line, but the recipient must be poor before and after the transfer. SDT allows either the donor or the recipient to cross the poverty line, but not both.

3.4 Symmetry axioms

Under this heading, the following axioms are included:

- **scale invariance**
- **translation invariance**
- **principle of population**

Scale invariance (SI) – This axiom requires that the invariance of the poverty index if **all incomes** of poor individuals **and the poverty line** be scaled by the same factor. A poverty index satisfying this axiom is called a **relative poverty index**.

Translation invariance (TI) – This axiom requires that the invariance of the poverty index, when all incomes of poor individuals and the poverty line be either increased or decreased by the same absolute amount. A poverty index satisfying this axiom is called an **absolute poverty index**.

Principle of population (PP) – This axiom requires that the poverty index be invariable when identical populations are replicated and pooled. In poverty analysis, this axiom is sometimes called **Size Independence Axiom** (SIN).

4 AXIOMS AND POVERTY MEASURES: AN EXAMPLE

In order to test how poverty indexes discussed in other modules may react to these axioms and to have an overall view of this problem, we will define a simple comprehensive example. Let us assume a population of ten men and, possibly, one woman as in Table 1, below. The first row of the table is the benchmark income

distribution, expressed, let's say, in $/year. The poverty line, reported in the far right column, is $ 2,000 /year.

In order to test the behaviour of poverty indexes with regard to the different axioms, the benchmark income distribution has been changed in the direction dictated by each axiom:

- F axiom is modelled by increasing incomes above the poverty line by $100/year;
- GF axiom is modelled by increasing incomes above the poverty line by $100/year and adding a non-poor person (Mary);
- SM is modelled by lifting Charles out of poverty;
- WM is modelled by increasing Charles' income without lifting him out of poverty;
- MT is modelled assuming a progressive transfer of 100 from Charles to Frank, both below the poverty line;
- WT is modelled by assuming a progressive transfer from David (above the poverty line both before and after the transfer) to Frank;
- SUT is modelled by a regressive transfer of 200 from Jack to Charles, who is lifted out of poverty;
- SDT is modelled by a progressive transfer of 200 from Robert to Charles, who is lifted out of poverty;
- SI is obviously modelled by increasing all incomes and the poverty line by 10 per cent;
- TI is modelled by increasing all incomes and the poverty line by $100/year.

Table 1 - Axioms and income distributions

	Jack	Frank	George	Paul	Charles	David	Robert	Mark	John	Timothy	Mary	Poverty line
Benchmark income distribution ($/year)	546	674	802	1467	1900	2320	2774	2987	2991	3076		2000
F: increase all non-poor incomes by 100	546	674	802	1467	1900	**2420**	**2874**	**3087**	**3091**	**3176**		2000
GF: increase all non-poor incomes by 100 and adding a non-poor income	546	674	802	1467	1900	**2420**	**2874**	**3087**	**3091**	**3176**	**4000**	2000
SM: lifting out of poverty	546	674	802	1467	**2001**	2320	2774	2987	2991	3076		2000
WM: poor income increase, no lifting out	546	674	802	1467	**1990**	2320	2774	2987	2991	3076		2000
MT: progressive transfer of 100, both below the poverty line	546	**774**	802	1467	**1800**	2320	2774	2987	2991	3076		2000
WT: progressive transfer of 100 from above the poverty line	546	**774**	802	1467	1900	**2220**	2674	2987	2991	3076		2000
SUT: regressive transfer of 200, recipient lifted out the poverty line	**346**	674	802	1467	**2100**	2320	2774	2987	2991	3076		2000
SDT: progressive transfer of 200, recipient lifted out the poverty line	546	674	802	1467	**2100**	2320	**2574**	2987	2991	3076		2000
SI: all incomes and the poverty line increased by 10 per cent	**601**	**741**	**882**	**1614**	**2090**	**2552**	**3051**	**3286**	**3290**	**3384**		**2200**
TI: all incomes and the poverty line increased by 100	**646**	**774**	**902**	**1567**	**2000**	**2420**	**2874**	**3087**	**3091**	**3176**		**2100**

Applying axioms give rise to different income distributions in Table 1. It is therefore interesting to understand how different poverty indexes should behave, according to each axiom, and how they actually behave. Table 2, below, illustrates this behaviour.

Table 2 - Axioms and poverty indexes: An example

	Required behaviour of the poverty index	HC	PG	S	FGT (0.5)	FGT (2)	KA (0.5)	KA (2)	TH	SK	BD (0.5)	BD (2)	TA
Benchmark income distribution ($/year)	Benchmark	0.500	0.461	0.289	0.318	0.140	0.263	0.321	0.367	0.444	0.202	0.265	0.350
F: increase all non-poor incomes by 100	Invariance	0.500	0.461	0.289	0.318	0.140	0.263	0.321	0.367	0.444	0.202	0.265	0.350
GF: increase all non-poor incomes by 100 and adding a non-poor income	Invariance	0.500	0.461	0.263	0.289	0.127	0.239	0.292	0.341	0.444	0.202	0.265	0.269
SM: lifting out of poverty	Decrease	0.400	0.564	0.254	0.296	0.140	0.242	0.270	0.362	0.370	0.219	0.236	0.225
WM: poor income increase, no lifting out	Decrease	0.500	0.452	0.287	0.303	0.140	0.261	0.320	0.362	0.442	0.183	0.264	0.354
MT: progressive transfer of 100, both below the poverty line	Decrease	0.500	0.461	0.284	0.324	0.134	0.260	0.314	0.364	0.448	0.210	0.259	0.350
WT: progressive transfer of 100 from above the poverty line	Decrease	0.500	0.451	0.282	0.315	0.134	0.257	0.314	0.359	0.445	0.198	0.259	0.354
SUT: regressive transfer of 200, recipient lifted out the poverty line	Increase	0.400	0.589	0.270	0.301	0.155	0.255	0.291	0.380	0.366	0.227	0.249	0.282
SDT: progressive transfer of 200, recipient lifted out the poverty line	Decrease	0.400	0.564	0.254	0.296	0.140	0.242	0.270	0.362	0.370	0.219	0.236	0.284
SI: all incomes and the poverty line increased by 10 per cent	Invariance	0.500	0.461	0.289	0.318	0.140	0.263	0.321	0.367	0.444	0.202	0.265	0.350
TI: all incomes and the poverty line increased by 100	Invariance	0.500	0.439	0.275	0.310	0.127	0.251	0.306	0.350	0.446	0.193	0.252	0.359

Note: HC=Headcount ratio; PG=Poverty gap; S=Sen index; FGT=Foster, Greer and Thorbecke index; KA=Kakwani index; TH=Thon index; SK=Sen-Kakwani index; BD=Blackorby-Donaldson index; TA=Takayama index.

The first column repeats the axiom we are dealing with, while the second column gives the required behaviour of the poverty index dictated by each axiom. All other columns report the values of each poverty index calculated on the income distributions as modified in Table 1, above. The shaded boxes in Table 2, above, reports all those cases where the corresponding poverty index (on the columns) violates the axiom (on the rows). As can be easily seen, violations are many. In particular, there is no poverty index that satisfies all axioms at the same time. This suggests that the choice of the poverty index should be driven, to some extent, by those desirable properties we would like the index to respect.

Table 3, below, sums up the behaviour of poverty index, based on the previous example, highlighting the fact that no poverty index satisfies all axioms at the same time.

Table 3 - Axioms and poverty indexes: a synthesis

	HC	PG	S	FGT	KA	TH	SK	BD	TA
FOCUS									
Focus (FO)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Generalized focus (GF)	No	Yes	No	No	No	No	Yes	Yes	No
MONOTONICITY									
Strong (SM)	Yes	No	Yes	Yes, if a>0	Yes	Yes	Yes	No	Yes
Weak (WM)	No	Yes	Yes	Yes, if a>0	Yes	Yes	Yes	Yes	No
TRANSFER									
Minimal (MT)	No	No	Yes	No	Yes	Yes	No	No	No
Weak (WT)	No	Yes	Yes	Yes, if a>1	Yes	Yes	No	Yes	No
Strong upward (SUT)	No	Yes	No	No	No	Yes	No	Yes	No
Strong downward (SDT)	No	Yes	Yes	Yes, if a>1	Yes	Yes	Yes	Yes	Yes
SYMMETRY									
Scale invariance (SI)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Translation invariance (TI)	Yes	No	No	No	No	No	No	No	No
Population principle (PP-SIN)	Yes	Yes	No	Yes	No	No	No	No	No

Note: HC=Headcount ratio; PG=Poverty gap; S=Sen index; FGT=Foster, Greer and Thorbecke index; KA=Kakwani index; TH=Thon index; SK=Sen-Kakwani index; BD=Blackorby-Donaldson index; TA=Takayama index.

5 CONCLUSIONS

The basic result of this module is that poverty measures should respect some desirable properties. The most common desirable properties have been discussed and the behaviour of poverty indexes investigated. There is no single poverty measure that respects all axioms at the same time. Choosing a poverty index therefore means choosing a particular set of desirable properties to which to adhere.

6 READERS' NOTES

6.1 Time requirements

The delivery of this module to an audience already familiar with the definition of poverty may take up to two hours.

6.2 Frequently asked questions

- ✓ **How do we select among so many poverty indexes?** Poverty indexes are many, but they react differently to changes in the income distribution. Axioms may help to pre-define what characteristics a poverty index should have in the specific context and to select among poverty measures.
- ✓ **If recourse is made to axioms, are results in terms of poverty measures robust?** Axioms can only help us to select poverty measures. They do not provide robustness in themselves. Robustness may only be derived by experimenting different poverty measures with a fixed poverty line or a fixed poverty measure with a variable poverty line. Dominance may be more useful in help us find robust results.

- ✓ **What are the most important axioms?** It is difficult to set a priority scale on axioms. In applied works, however, one of the most important property of the poverty measures is the sensitivity to income transfers. The transfer axioms should therefore be considered with particular care.

7 REFERENCES AND FURTHER READINGS

A particularly useful paper on axioms for poverty measurement is:

Seidl C., 1988. Poverty Measurement: A Survey, in Bos D., Rose M., Seidl C. (eds), *Welfare and* Efficiency *in Public Economics*, Berlin and Tokyo, Springer, pp. 71-147. New York, USA.

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4. Summary

This module illustrates on what basis a poverty index should be chosen. In particular, it discusses the desirable properties a poverty index should respect. This desirable properties are called **axioms**. Four classes of axioms will be discussed: i) focus axioms; ii) monotonicity axioms; iii) transfer axioms; iv) symmetry axioms.

As we will see, there is no single measure of poverty respecting all axioms at the same time. Choosing a poverty measure may therefore imply a trade-off between different aims.

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6. Author(s)

Lorenzo Giovanni Bellù, Agricultural Policy Support Service, Policy Assistance Division, FAO, Rome, Italy
Paolo Liberati, University of Urbino, "Carlo Bo", Institute of Economics, Urbino, Italy

7. Module type	☐ Thematic overview ☐ Conceptual and technical materials ☒ Analytical tools ☐ Applied materials ☐ Complementary resources
8. Topic covered by the module	☒ Agriculture in the macroeconomic context ☐ Agricultural and sub-sectoral policies ☐ Agro-industry and food chain policies ☐ Environment and sustainability ☐ Institutional and organizational development ☐ Investment planning and policies ☒ Poverty and food security ☐ Regional integration and international trade ☐ Rural Development
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